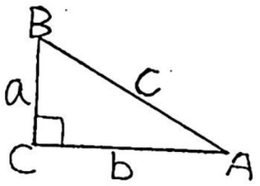
ピタゴラスの定理

生徒の作った「単元のまとめ」

三平方のピタゴラスの定理



∠C=∠R ならば

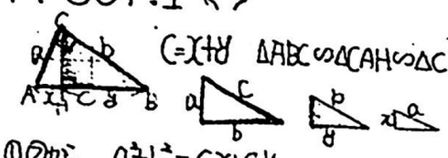
$a^2+b^2=c^2$

$c=\sqrt{a^2+b^2}$

定理

証明

proof.1

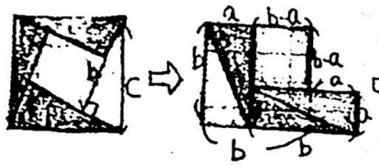


①②から $a^2+b^2=cx+cy$

$=c(x+y)$

$a^2+b^2=c^2$

proof.2



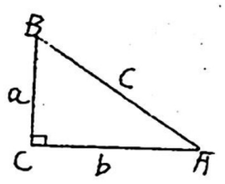
$\Rightarrow a^2+b^2=c^2$

$c^2=\frac{ab}{2}\times4+(b-a)^2$

$c^2=2ab+b^2-2ab+a^2$

$c^2=a^2+b^2$

三平方のピタゴラスの定理の逆

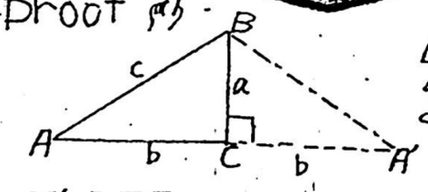


$a^2+b^2=c^2$ ならば

∠C=∠R

証明

proof



△ABCの反対側に∠BCA'が∠Rになるような三角形をかく AC=A'C

$BA'=\sqrt{a^2+b^2}$ $AB=\sqrt{a^2+b^2}$ AB=A'B=c

AC=A'C BC=BC AB=A'B

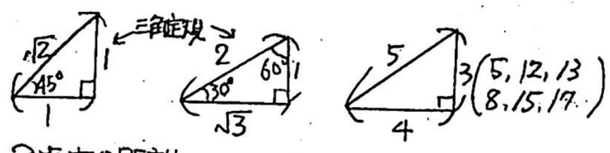
△ABC≡△BCA' ∠ACB=∠A'CB

∠A'CB=∠R より ∠C=∠R

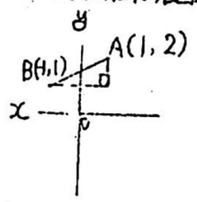
注

★=定理・重要 ▲=証明・重要

(1) 特殊な三角形の辺の比



(2) 2点間の距離



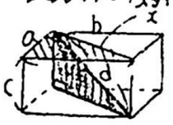
$AB^2=\{1-(-1)\}^2+(2-1)^2$

$=4+1$

$=5$

$AB=\sqrt{5}$

(3) 直方体の対角線



$x^2=a^2+b^2$

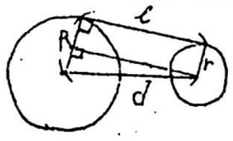
$x=\sqrt{a^2+b^2}$

$d^2=(\sqrt{a^2+b^2})^2+c^2$

$d^2=a^2+b^2+c^2$

$d=\sqrt{a^2+b^2+c^2}$

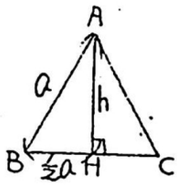
(4) 円の共通接線の長さ



$\ell^2=d^2-(R-r)^2$

$\ell=\sqrt{d^2-(R-r)^2}$

(5) 正三角形の高さ



$h^2=a^2-(\frac{1}{2}a)^2$

$h^2=a^2-\frac{a^2}{4}$

$h^2=\frac{3}{4}a^2$

$h=\frac{\sqrt{3}}{2}a$

(6) 垂形の高さ

例 正四角垂

$AH^2=e^2-r^2$

$AH=\sqrt{e^2-r^2}$

$h^2=a^2-(\sqrt{e^2-r^2})^2$

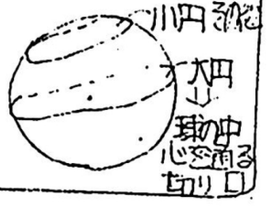
$h^2=a^2-(e^2-r^2)$

$h=\sqrt{a^2-(e^2-r^2)}$

(7) 弧が一定の時 中心から弦まではいつでも等しい

$d^2=r^2-x^2$

$d=\sqrt{r^2-x^2}$



感想

- ピタゴラスの定理の 応用の広さに 何か 不思議を感じた。
- よくわかったような わからないような中途半ぱ
- 前向く態度が 悪かった

[redacted]